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Fuzzy multi-choice goal programming

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ABSTRACT

Goal programming as a well known technique has been widely used for solving multi objective decision making problems. However, in some practical cases, there may exist situations where the decision maker is interested in setting multi aspiration levels for objectives that may not be expressed in a precise manner. In this paper, a novel formulation of fuzzy multi-choice goal programming (FMCGP) is presented. The proposed approach not only improves the applicability of goal programming in real world situations but also provides useful insight about the solution of a new class of problems. To illustrate and clarify the proposed approach, a numerical example is presented.

1. Introduction

Goal programming (GP) as one of the most used and well known decision making techniques, was introduced by Charnes and Cooper [1] in 1961. The interesting philosophy and high applicability of GP in handling real world decision making problems with multi objectives structures made it very useful and widespread. This leads to further development of GP for different decision making problems. The related research can be categorized into two broad classes: goal programming techniques which are proposed for crisp decision making problems and fuzzy goal programming models. Most research in the goal programming literature belongs to the first class. The research by Lee [2], Ignizio [3], Romero [4], and Tamiz et al. [5] belongs to this class. Schniederjans [6] provided a bibliography of researches relating to goal programming till 1995 and Jones and Tamiz also presented a bibliography of the related researches published during 1990–2000.

The second class includes the developed goal programming models for decision making in fuzzy environment. The proposed models in this category used the fuzzy set theory as a modeling tool to deal with the uncertainty of real world decision making problems. The uncertainty of decision making problem may exist because of imprecision aspiration levels, using linguistic variables, vague objective priorities or weights, uncertainty of resources, technological coefficients, etc. In the 1980s, fuzzy sets have been used in GP models to deal with the uncertainty of parameter and as well to represent a satisfaction degree of the decision maker with respect to his/her preference structure. Research by Narasimhan [7], Hannan [8,9], Tiwari et al. [10], Mohamed [11], Wang and Fu [12], Chen and Tsai [13] and many others are researches that belong to the second class. A survey of various fuzzy goal programming (FGP) models can be found in Chanas and Kuchta [14]. Finally, a comprehensive overview of the state-of-the-art in goal programming can be found in [15,16].

In the reviewed models, mostly classical structure is used and the general structure of goal programming models, including crisp and fuzzy ones, remained unchanged. However, in the real world situations, decision making problems may arise with different structures which can not be handled using standard decision making approaches. For example when in a multi objective decision making problem, the decision maker presents multi aspiration levels as goals for each objective, the classical models of decision making including goal programming can not be applied directly. To deal with this type of

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problems, it is essential to develop new decision making models. To do this, Chang [17] proposed a multi-choice goal programming (MCGP) approach to deal with such problems. He revised his approach [18] to make easier understanding and implementation of linear programming packages for solving such problems. Liao [19] also presented the formulation of multi-segmented goal programming which can be applied to solve multiple decision making problems which have multi-segmented aspiration levels. As seen, the last three researches are conducted to solve multiple decision making problems that more than one aspiration level is chosen by the decision maker. These researches can be categorized into the crisp decision making models; however, in the real life decision making problems it may exist situations that the decision maker could not or he/she is not interested in presenting his/her preferred aspiration levels as specified and crisp values. This may occur because of complexity or imprecise nature of situation or imprecise preferences of the decision maker. To the best of our knowledge, there are no decision making model for solving this type of problems. Therefore, in this paper, the fuzzy multi-choice goal programming (FMCGP) approach is formulated and used to solve this problem. The proposed approach can be widely used in the corresponding real world problems in an efficient manner.

The remainder of paper is organized as follows: in the next section, the formulation of FMCGP is proposed. In Section 3, an illustrative example is provided to show applicability of proposed method. Finally, conclusion is presented.

2. Fuzzy multi-choice goal programming formulation

For the first time in goal programming literature, Chang [17] proposed MCGP approach which allows DMs to set multi-choice aspiration levels (MCAL) for each goal (i.e., one goal mapping multiple aspiration levels). However, in some cases authors believe that these aspiration levels can be imprecise or fuzzy (i.e., each goal mapping many fuzzy aspiration levels).To the best of our knowledge, no work has been done for solving this typical FMCGP problem. Generally, a FMCGP problem can be formulated as follows:

$$\begin{aligned} &\text{Min} \quad \sum_{i=1}^{n} w_i |f_i(X) - \tilde{g}_{i1} \text{ or } \tilde{g}_{i2} \text{ or } \dots \text{ or } \tilde{g}_{im}| \\ &\text{s.t.} \qquad X \in F \quad (F \text{ is a feasible set}), \end{aligned} \tag{1}$$

where w_i, $i = 1, \ldots, n$ are the relative importance of objective function and the aspiration levels $\tilde{g}_{ik}$, $K = 1, \ldots, m$ are assumed to be triangle fuzzy numbers with membership function μ_{ik}, $K = 1, \ldots, m$. In order to solve the above mentioned problem, the Zimmerman's approach [20] is used to transform (1) into a conventional linear programming problem. For clarifying the FMCGP problem, let us consider three cases of decision problems in Figs. 1–3, respectively. We assume that $G_k(x)$ is the kth objective function.

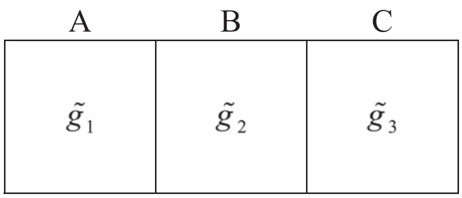

Fig. 1. Example of FMCGP (one fuzzy aspiration level for each goal).

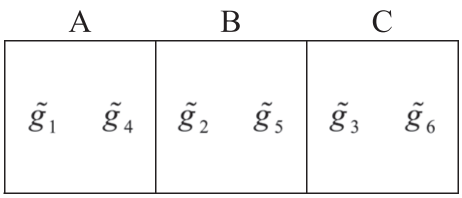

Fig. 2. Example of FMCGP (two fuzzy aspiration levels for each goal).

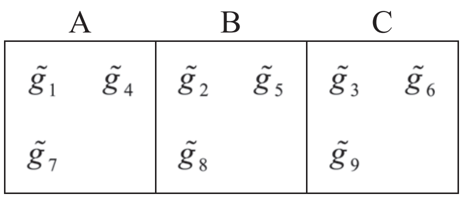

Fig. 3. Example of FMCGP (multi fuzzy aspiration levels for each goal).

(i) Assuming only one fuzzy aspiration level for each goal. For example, there are three fuzzy aspiration levels $\tilde{g}_1, \tilde{g}_2$ and $\tilde{g}_3$ corresponding to goals A, B and C (see Fig. 1). This case can be modeled as follows:

$$\begin{array}{ll} \text{Maximize} & \mu_1+\mu_2+\mu_3 \\ \text{s.t.} & \mu_1 \leqslant 1-\frac{G_1(x)-\tilde{g}_1}{d_1^-}, \\ & \mu_1 \leqslant 1-\frac{\tilde{g}_1-G_1(x)}{d_1^+}, \\ & \mu_2 \leqslant 1-\frac{G_2(x)-\tilde{g}_2}{d_2^-}, \\ & \mu_2 \leqslant 1-\frac{\tilde{g}_2-G_2(x)}{d_2^+}, \\ & \mu_3 \leqslant 1-\frac{G_3(x)-\tilde{g}_3}{d_3^-}, \\ & \mu_3 \leqslant 1-\frac{\tilde{g}_3-G_3(x)}{d_3^+}, \\ & \mu_1,\mu_2,\mu_3 \geqslant 0, \\ & X \in F \quad (F \text{ is a feasible set}), \end{array} \qquad \text{Model (1)}$$

where d_i^+ and d_i^- $(i{:}1,2,3)$ are the maximum allowable negative and positive deviations from $\tilde{g}_i$.

(ii) Two fuzzy aspiration levels for each goal. This is a case of FGP with an either-or selection. The target in goal A is to choose an appropriate fuzzy aspiration level from either $\tilde{g}_1$ or $\tilde{g}_4$, while the target in goal B is to choose an appropriate fuzzy aspiration level from either $\tilde{g}_2$ or $\tilde{g}_5$, and also the target in goal C is to choose an appropriate fuzzy aspiration level from either $\tilde{g}_3$ or $\tilde{g}_6$ (see Fig. 2). Based on the modeling of the Chang [17], three extra binary variables should be added as described below.

$$\begin{array}{ll} \text{Maximize} & \mu_1+\mu_2+\mu_3 \\ \text{s.t.} & \mu_1 \leqslant 1-\left[\frac{G_1(x)-\tilde{g}_1}{d_{11}^-}z_1+\frac{G_1(x)-\tilde{g}_4}{d_{14}^-}(1-z_1)\right], \\ & \mu_1 \leqslant 1-\left[\frac{\tilde{g}_1-G_1(x)}{d_{11}^+}z_1+\frac{\tilde{g}_4-G_1(x)}{d_{14}^+}(1-z_1)\right], \\ & \mu_2 \leqslant 1-\left[\frac{G_2(x)-\tilde{g}_2}{d_{22}^-}z_2+\frac{G_2(x)-\tilde{g}_5}{d_{25}^-}(1-z_2)\right], \\ & \mu_2 \leqslant 1-\left[\frac{\tilde{g}_2-G_2(x)}{d_{22}^+}z_2+\frac{\tilde{g}_5-G_2(x)}{d_{25}^+}(1-z_2)\right], \\ & \mu_3 \leqslant 1-\left[\frac{G_3(x)-\tilde{g}_3}{d_{33}^-}z_3+\frac{G_3(x)-\tilde{g}_6}{d_{36}^-}(1-z_3)\right], \\ & \mu_3 \leqslant 1-\left[\frac{\tilde{g}_3-G_3(x)}{d_{33}^+}z_3+\frac{\tilde{g}_6-G_3(x)}{d_{36}^+}(1-z_3)\right], \\ & 0 \leqslant \mu_1,\mu_2,\mu_3 \leqslant 1, \\ & X \in F \ (F \text{ is a feasible set}) \end{array} \qquad \text{Model (2)}$$

where z_1, z_2 and z_3 are binary variables and d_{ij}^+ and d_{ij}^- $(i{:}1,2,3, j{:}1,2)$ are the maximum allowable negative and positive deviations from the jth fuzzy aspiration level in the ith goal, respectively.

(iii) Multi fuzzy aspiration levels for each goal. This is a case of FGP with multi-choice selection. The target in goal A is to choose an appropriate fuzzy aspiration level between $\tilde{g}_1, \tilde{g}_4$ and $\tilde{g}_7$, while the target in goal B is to choose an appropriate fuzzy aspiration level between $\tilde{g}_2, \tilde{g}_5$ and $\tilde{g}_8$, and the target in goal C is to choose an appropriate fuzzy aspiration level between $\tilde{g}_3, \tilde{g}_6$ and $\tilde{g}_9$ (see Fig. 3). Based on the modeling of the Chang [17], six extra binary variables should be added as described below.

$$\begin{array}{ll} \text{Maximize} & \mu_1+\mu_2+\mu_3 \\ \text{s.t.} & \mu_1 \leqslant 1-\left[\frac{G_1(x)-\tilde{g}_1}{d_{11}^-}z_1z_2+\frac{G_1(x)-\tilde{g}_4}{d_{14}^-}z_1(1-z_2)+\frac{G_1(x)-\tilde{g}_7}{d_{17}^-}z_2(1-z_1)\right], \\ & \mu_1 \leqslant 1-\left[\frac{\tilde{g}_1-G_1(x)}{d_{11}^+}z_1z_2+\frac{\tilde{g}_4-G_1(x)}{d_{14}^+}z_1(1-z_2)+\frac{\tilde{g}_7-G_1(x)}{d_{17}^+}z_2(1-z_1)\right], \\ & \mu_2 \leqslant 1-\left[\frac{G_2(x)-\tilde{g}_2}{d_{22}^-}z_3z_4+\frac{G_2(x)-\tilde{g}_5}{d_{25}^-}z_3(1-z_4)+\frac{G_2(x)-\tilde{g}_8}{d_{28}^-}z_4(1-z_3)\right], \\ & \mu_2 \leqslant 1-\left[\frac{\tilde{g}_2-G_2(x)}{d_{22}^+}z_3z_4+\frac{\tilde{g}_5-G_2(x)}{d_{25}^+}z_3(1-z_4)+\frac{\tilde{g}_8-G_2(x)}{d_{28}^+}z_4(1-z_3)\right], \\ & \mu_3 \leqslant 1-\left[\frac{G_3(x)-\tilde{g}_3}{d_{33}^-}z_5z_6+\frac{G_3(x)-\tilde{g}_6}{d_{36}^-}z_5(1-z_6)+\frac{G_3(x)-\tilde{g}_9}{d_{39}^-}z_6(1-z_5)\right], \\ & \mu_3 \leqslant 1-\left[\frac{\tilde{g}_3-G_3(x)}{d_{33}^+}z_5z_6+\frac{\tilde{g}_6-G_3(x)}{d_{36}^+}z_5(1-z_6)+\frac{\tilde{g}_9-G_3(x)}{d_{39}^+}z_6(1-z_5)\right], \\ & z_1+z_2 \geqslant 1, \\ & z_3+z_4 \geqslant 1, \\ & z_5+z_6 \geqslant 1, \\ & 0 \leqslant \mu_1,\mu_2,\mu_3 \leqslant 1, \\ & X \in F \ (F \text{ is a feasible set}) \end{array} \qquad \text{Model (3)}$$

where z_1, z_2, z_3, z_4, z_5 and z_6 are binary variables and d_{ij}^{+} and d_{ij}^{-} (i:1,2,3, j:1,2,3) are the maximum allowable negative and positive deviations from the jth aspiration level in the ith goal, respectively.

According to Chang [17], the quadratic terms z_1z_2, z_3z_4 and z_5z_6 can be transformed to linear form as follows.

Let $x = z_iz_j$, where x satisfies the following inequalities:

$$(z_i + z_j - 2) + 1 \leqslant x \leqslant (2 - z_i - z_j) + 1, \tag{2}$$

$$x \leqslant z_i, \tag{3}$$

$$x \leqslant z_j, \tag{4}$$

$$x \geqslant 0, \tag{5}$$

The above inequalities can be checked as follows:

(i) if $z_i = z_j = 1$ then $x = 1$ (from (2)).
(ii) if $z_iz_j = 0$ then $x = 0$ (from (3)–(5)).

Let $G_k(x)$ denote the kth objective function. As mentioned before, the linear membership function μ_i for the ith fuzzy goal can be defined as

$$\mu_i = \begin{cases} 0 & \text{if} \quad G_i(x) \geqslant \tilde{g}_{ij} + d_{ij2}, \\ 1 - \sum_{j=1}^{m} \frac{G_i(x)-\tilde{g}_{ij}}{d_{ij}^{-}} S_{ij}(B) & \text{if} \quad \tilde{g}_{ij} \leqslant G_i(x) \leqslant \tilde{g}_{ij} + d_{ij2}, \\ 1 & \text{if } G_i(x) = \tilde{g}_{ij}, \qquad\qquad i: 1,2,\ldots,n, \\ 1 - \sum_{j=1}^{m} \frac{\tilde{g}_{ij}-G_i(x)}{d_{ij}^{+}} S_{ij}(B) & \text{if } \tilde{g}_{ij} - d_{ij1} \leqslant G_i(x) \leqslant \tilde{g}_{ij}, \\ 0 & \text{if otherwise}, \end{cases} \tag{6}$$

where $S_{ij}(B)$ represents a function of binary serial numbers that ensure only one aspiration level must be chosen in each goal; d_{ij}^{+} and d_{ij}^{-} are the maximum allowable negative and positive deviations from the jth aspiration level in the ith goal, respectively. Then the resulting FMCGP formulation can be expressed as follows:

$$\begin{aligned} &\text{Maximize} \quad f(\mu) = \sum_{i=1}^{n} w_i\mu_i \\ &\text{s.t.} \quad \mu_i \leqslant 1 - \sum_{j=1}^{m} \frac{G_i(x)-\tilde{g}_{ij}}{d_{ij}^{-}} S_{ij}(B), \quad i = 1,2,\ldots,n, \\ &\qquad \mu_i \leqslant 1 - \sum_{j=1}^{m} \frac{\tilde{g}_{ij}-G_i(x)}{d_{ij}^{+}} S_{ij}(B), \quad i = 1,2,\ldots,n, \\ &\qquad X \in F (F \text{ is a feasible set}), \\ &\qquad \mu_i \geqslant 0, \quad i = 1,2,\ldots,n. \end{aligned} \qquad \text{Model (4)}$$

In the following section, a numerical example is presented to illustrate the proposed approach.

3. An illustrative example

A company is manufacturing three products y_1, y_2 and y_3. For product y_1, there are three customers A, B and C with "approximate" demands 30, 50 and 70, respectively. The maximum allowable negative and positive deviation of customers A, B and C, from their goals are equal and set as 4, 5 and 6, respectively. The selling profit of this product is 10$. The information about these three products is shown in Table 1. However, because of some limitations such as political ones, the company has to select only one of its customers for each product. A profit of at least 850$ dollars from products' selling is expected. Three resources S_1, S_2 and S_3 are needed to produce these products. The amounts of each resource which is needed to produce each product are presented in Table 2.

This is a case of FMCGP which cannot be solved by current GP approaches.

For this problem, the related goals are listed below.

(G_1) $y_1 \simeq 30$ *or* 50 *or* 70,

(G_2) $y_2 \simeq 15$ *or* 30,

(G_3) $y_3 \simeq 30$ *or* 50.

Table 1
Related information about products.

Product	Customer	Demands	Maximum allowable neg. and pos. deviation	Profit ($)
y_1	A	30	4	10
	B	50	5	
	C	70	6	
y_2	D	15	3	12
	E	30	4	
y_3	F	10	2	15
	G	20	3	

Table 2
The amount of consumptions of resources for each product.

Resource	Product		
	y_1	y_2	y_3
S_1	5	7	4
S_2	3	5	6
S_3	1	2	1

Based on FMCGP method, assuming w_1, w_2 and w_3 are 0.4, 0.3 and 0.3. This problem is formulated as below

$$
\begin{aligned}
\text{Maximize}\quad & f(\mu) = 0.4\mu_1 + 0.3\mu_2 + 0.3\mu_3 \\
\text{s.t.}\quad & \mu_1 \leqslant 1 - \left[\tfrac{y_1-30}{4} z_1 z_2 + \tfrac{y_1-50}{5} z_1(1-z_2) + \tfrac{y_1-70}{6} z_2(1-z_1)\right], \\
& \mu_1 \leqslant 1 - \left[\tfrac{30-y_1}{4} z_1 z_2 + \tfrac{50-y_1}{5} z_1(1-z_2) + \tfrac{70-y_1}{6} z_2(1-z_1)\right], \\
& \mu_2 \leqslant 1 - \left[\tfrac{y_2-15}{3} z_3 + \tfrac{y_2-30}{4}(1-z_3)\right], \\
& \mu_2 \leqslant 1 - \left[\tfrac{15-y_2}{3} z_3 + \tfrac{30-y_2}{4}(1-z_3)\right], \\
& \mu_3 \leqslant 1 - \left[\tfrac{y_3-10}{2} z_4 + \tfrac{y_3-20}{3}(1-z_4)\right], \\
& \mu_3 \leqslant 1 - \left[\tfrac{10-y_3}{2} z_4 + \tfrac{20-y_3}{3}(1-z_4)\right], \\
& 10y_1 + 12y_2 + 15y_3 \geqslant 850, \\
& y_1 \leqslant \tfrac{x_{11}}{5}, \\
& y_1 \leqslant \tfrac{x_{12}}{3}, \\
& y_1 \leqslant x_{13}, \\
& y_2 \leqslant \tfrac{x_{21}}{7}, \\
& y_2 \leqslant \tfrac{x_{22}}{5}, \\
& y_2 \leqslant \tfrac{x_{23}}{2}, \\
& y_3 \leqslant \tfrac{x_{31}}{4}, \\
& y_3 \leqslant \tfrac{x_{32}}{6}, \\
& y_3 \leqslant x_{33}, \\
& x_{11} + x_{12} + x_{13} \leqslant 400, \\
& x_{21} + x_{22} + x_{23} \leqslant 380, \\
& x_{31} + x_{32} + x_{33} \leqslant 120, \\
& z_1 + z_2 \geqslant 1, \\
& 0 \leqslant \mu_1, \mu_2, \mu_3 \leqslant 1,
\end{aligned}
$$

To solve this problem, LINGO [21] is used. The optimal solution of the above mentioned problem is obtained as $(y_1, y_2, y_3, z_1, z_2, z_3, z_4) = (52.8, 13.2, 10.9, 1, 0, 1, 1)$ and resulting achievement degrees for three fuzzy goals μ_1, μ_2 and μ_3 are 0.44, 0.39 and 0.54, respectively.

As seen in the above example, the proposed approach can be used to deal with the multi objective programming problems with multiple fuzzy aspiration levels which can not be solved using conventional goal programming approach.

4. Conclusion

In this paper, a novel approach for solving fuzzy multi-choice goal programming problems was proposed. This method was the extension of multi-choice goal programming method for imprecise aspiration levels. The classical GP, fuzzy GP and MCGP are the special cases of proposed method. Finally, the applicability of this approach was illustrated using a numerical example. To deal with high level of uncertainty in the real world problems, the extension of proposed approach for multiple objective programming methods with uncertain possibilistic parameters can be considered as a future research.

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